CHARACTERISTICS OF AN UNSTABLY LOCALIZED DISTURBANCE IN A COMPRESSED BOUNDARY LAYER

A. S. Dryzhov

UDC 532.526

Experiments have demonstrated [1] that the transition of streamline-type flow into turbulent flow in a boundary layer occurs as a result of the formation and development of turbulent spots apparently arising from small natural disturbances. A study of the nonlinear evolution and interaction of localized disturbances requires knowledge of their characteristics to a linear approximation [2]. In the current work, results are presented of calculations of such characteristics for the first two unstable modes in a supersonic boundary layer on a two-dimensional plate ($M=4.5$, $T_W=4.44$).

The nature of the instability of a compressible boundary layer has been considered [3]. An analysis of the asymptotic ($t\to\infty$) behavior of the solution of the initial problem for a disturbance arising in a finite region in space also allows us to determine the characteristics of an unstable wave train. At high t, the solution of the initial problem is written in the form [3]

$$\psi=\sum_n\int d\mathbf{k}\psi_{nk}e^{p_n(\mathbf{k})t+i\mathbf{kr}}, \tag{1}$$

where Re $P_n(\mathbf{k})>0$ in some region of the k-plane, n is the number of the unstable mode, and $p_n(k)$ are determined from Eq. (3.11) [3].

Let us write the asymptotic equation for one term of the series (1) at high t using the method of steepest descent,

$$\psi=\frac{\text{const}}{t\sqrt{H}}\psi_{\mathbf{k}_s}e^{\Omega t}\{1+O(1/t)\}, \tag{2}$$

$$H=\left(\omega''_{\alpha\alpha}\omega''_{\beta\beta}-\omega''_{\alpha\beta}\right)_{\mathbf{k}=\mathbf{k}_s},\ \Omega=i\left(\mathbf{k}_s\mathbf{v}-\omega_s\right),$$

$$\mathbf{v}=\mathbf{r}/t,\ w=ip,\ \omega_s=\omega(\mathbf{k}_s),$$

where α and β are the components of the vector k in the x and z directions, respectively, and k_s is determined from the equation

$$\frac{d\omega}{d\mathbf{k}}=\mathbf{v}. \tag{3}$$

In the general case k_s is a complex vector and Eqs. (2) describe a packet with frequency $\omega=\text{Re}\,\omega_s(v)$, wave vector $\mathbf{k}=\text{Re}\,k_s(v)$, and space and time modulated amplitude.

Equation (3) was numerically solved for the first two unstable modes in the case of a two-dimensional disturbance, expanding at zero angle to the flow. The results of the calculations are depicted in Figs. 1-6. Figure 1 shows the location of the level line Re $\Omega=0$ for the second mode in the complex plane α for the values $v=0$, 0.5, 0.666, 0.83, and 1.0 (the corresponding level lines are indicated by the digits 1-5) when Re = 550. The dependence ReΩ(v) for the second mode when Re = 550, 1140, and 3000 are shown in Fig. 2 by solid curves (the corresponding curves are denoted by the letters a, b, and c). The same magnitude for the first mode (Re = 1600) is depicted by a broken line.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 50-53, January-February, 1976. Original article submitted February 11, 1975.

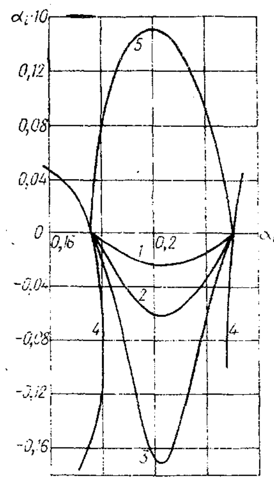

Fig. 1

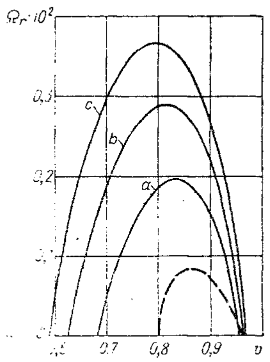

Fig. 2

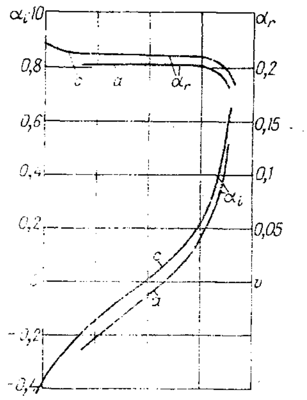

Fig. 3

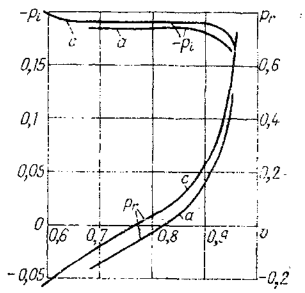

Fig. 4

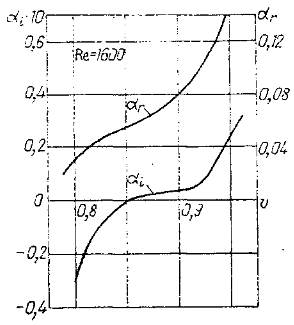

Fig. 5

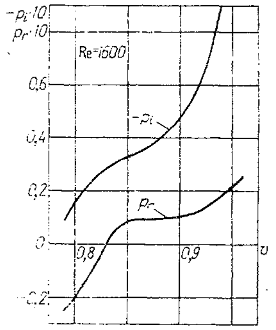

Fig. 6

Clearly, Re $\Omega > 0$ in some range of values of v, i.e., the localized disturbance decays into unstable trains propagating downstream at a finite velocity. The most rapidly intensified component of the train will move at a velocity

$$v_g = \left(\frac{d\omega_r}{d\alpha_r}\right)_{\alpha=\alpha_0},$$

where α_0 is a point on the real α axis at which ω_i reaches a maximum. (The subscripts "r" and "i" denote the real and imaginary parts of the corresponding variables). Figures 3 and 4 depict the spectral characteristics for the second mode (curves a and c for Re = 550 and 3000) and Figs. 5 and 6 for the first mode. The wave number and frequency for the second mode are nearly constant within the entire range of variation of b, such that $\Omega_r > 0$, i.e., the unstable train associated with the second mode is nearly monochromatic. The wave number and frequency for the first mode in the range of intensification vary greatly.

The pattern of convective hydrodynamic instability is qualitatively simulated in a linear approximation by an equation with constant coefficients,

$$\psi_t + u\psi_x - (i\varepsilon + \xi)\psi_{xx} + i\xi(\alpha_1 + \alpha_2)\psi_x + \alpha_1\alpha_2\psi = 0, \tag{4}$$

whose dispersion equation has the form

$$p(\alpha) = -i(u\alpha + \varepsilon\alpha^2) - \xi(\alpha - \alpha_1)(\alpha - \alpha_2). \tag{5}$$

All the constants in Eq. (4) are real and u, ξ, α_1, and α_2 are positive.

The solution of Eq. (3) for the dispersion equation (5) is expressed in the form

$$\alpha_s = \alpha_0 + \frac{(v - v_g)(\varkappa + i)}{2\lambda}, \quad v = x/t,$$

$$\varkappa = \varepsilon/\xi, \ \lambda = \xi(1 + \varkappa^2), \ \alpha_0 = (\alpha_1 + \alpha_2)/2, \ v_g = u + 2\varepsilon\alpha_0;$$

$$\Omega_r(\alpha_s) = \xi\frac{(\alpha_2-\alpha_1)^2}{4} - \frac{(v-v_g)^2}{4\lambda};\ \Omega_i(\alpha_s) = \alpha_0(v-v_f) + \varkappa\frac{(v-v_g)^2}{4\lambda},\ v_f = u + \varepsilon\alpha_0.$$

Equation (5) for the second mode yields a satisfactory quantitative approximation. The constants in Eq. (4) for Re = 3000 have the values

$$\alpha_1 \approx 0.186;\ \ \alpha_2 \approx 0.243;\ \ \xi \approx 4.5;\ \ \varepsilon \approx -0.515;\ \ u \approx 1.02.$$

LITERATURE CITED

1. H. Schlichting, Boundary Layer Theory, McGraw-Hill (1966).
2. K. Stewartson, and J. T. Stuart, "A nonlinear instability theory for a wave system in plane Poiseuille flow," J. Fluid Mech., 48, Part 3 (1971).
3. A. S. Dryzhov, "Stability of a compressible boundary layer relative to a localized disturbance," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1973).

CALCULATION OF THE INTERACTION OF A TURBULENT BOUNDARY LAYER WITH AN EXTERNAL SUPERSONIC FLOW ON THE CONCAVE CORNER AND ON THE SPHERICAL INTAKE PART OF A BODY

A. N. Antonov

UDC 532.526.4 : 533.6.011.5

INTRODUCTION

An integrated method of calculating turbulent flow on two-dimensional and axisymmetrical bodies in separation and attached boundary layer zones arising in the neighborhood of a concave corner and on a spherical intake part of a body is proposed. The method allows us to calculate pressure distribution, displacement thickness, and momentum thickness within the region in which the boundary layer interacts with an external ideal flow. The phenomenon of the interaction between a viscous and nearly inviscous flow is widespread. It is observed when a concave corner is streamlined, as a pressure shock impinges on a boundary layer, in the case of flow in the neighborhood of the spherical intake part of an axisymmetrical body, and in many other cases. The distinctive features of this phenomenon when two-dimensional and axisymmetrical bodies are streamlined has been theoretically investigated in [1-4]. Separated flows due to a pressure shock or an obstacle have been studied in [1-3], while [4] determined the base pressure behind the spherical intake part of a body. Theoretical investigations for the case of "free" separated flows in which the separation point and the attached boundary layer were not fixed, for example, on a plate with long wedge attached to it, have been carried out within the context of boundary-layer theory using integrated methods. In the current article, an integrated method of calculating flows in a base region [5] is used to calculate "free" separated flows in the neighborhood of a concave corner and on a spherical intake part of a body with a base support. The results of the calculations are compared to experimental data.

§1. Let us consider the following approximate flow scheme in the separation zones of a boundary layer in front of a wedge (flap) in the form of a scheme for the ordinary interaction of a turbulent boundary layer with an external ideal flow (Fig. 1). The interaction region is within the separation zone 1-4 and the attached zone 5-8.

In the separation zone, we distinguish gradient flow 1-3 and constant-pressure flow 3-4; S_1S_2 is the constant flow rate line, where S_1 and S_2 are critical points. The calculation of the interaction of viscous layers

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 53-61, January-February, 1976. Original article submitted November 18, 1974.